Drop breakup in the flow through fixed fiber beds: An experimental and computational investigation

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Dilute fixed fiber beds provide a model system for studying drop dynamics in disordered flows. Fluctuations about the mean uniform velocity are generated by fiber elements within the media, and the disturbance velocities far from any single fiber (at distances on the order of the pore size) have been predicted to be strong in terms of drop deformation and breakup by Mosler and Shaqfeh [Phys. Fluids 9, 5 (1997)]. In this work, we focus on the importance of near-field interactions, or the flow close to individual fibers. We present experimental observations of drop deformation and breakup during flow through a dilute bed of randomly placed fibers. We found breakup to result from only close interactions with fibers and describe two near-field breakup mechanisms which we term “graze” and “hairpin” processes. In addition, we present the breakup probability through the experimental fiber bed as a function of the appropriate Capillary number Ca. To better understand the near-field interactions, we used the boundary integral method to determine drop shape evolution in the flow around an infinite fiber within a porous medium, and our simulations capture the breakup mechanisms observed during experiments. To compare with experimental breakup probabilities, we have defined a critical offset for breakup during flow past a fiber and assuming straight center-of-mass trajectories, calculated breakup probabilities based on this simple model. These predictions compare well with the experimental measurements for Ca ≥ 2. © 2003 American Institute of Physics. [DOI: 10.1063/1.1557051]

I. INTRODUCTION

As drops are carried by a suspending fluid through a fixed bed of randomly positioned fibers, they experience a Lagrangian-unsteady flow under creeping flow conditions. Even though the mean flow through the bed is uniform and steady, previous work on complex fluids in dilute beds has shown interesting microstructural behavior attributable to fluctuations about the mean velocity. Experimental and theoretical studies have characterized these fiber bed flows as being strong in terms of orienting tracer particles, extending polymer molecules, and breaking drops. In this work, our main objective is to understand drop breakup mechanisms by using both experimental and computational methods to focus on the near-field interactions between drops and individual fibers within the bed.

In addition to providing insight into the behavior of emulsions in porous media, flows through fixed fiber beds are a model system for furthering our understanding of drop deformation and breakup within time-varying flows. Eulerian and Lagrangian unsteady flows are present in numerous industrial processes (e.g., fluid circulation, mixing, filtration, oil recovery), and these flows can impact the microstructure of emulsions (e.g., oil-in-water, polymer blends, biological fluids) by causing deformation and breakup. In turn, changes in microstructure (in terms of drop shape and size) have strong implications on rheological properties, emulsion stability, and heat and mass transfer. To better understand the coupling between viscous forces and drop dynamics in complicated industrial applications, past research has focused on the dynamics of Newtonian drops under Stokes flow.

Taylor first showed experimentally that steady linear flows can deform spherical drops into steady ellipsoidal shapes. For small deformations, he developed a perturbation theory relating drop shapes to the Capillary number Ca (ratio of viscous forces to surface tension) and the ratio between drop fluid and suspending fluid viscosities. For each flow type and viscosity ratio, Taylor experimentally measured a critical Capillary number for drop breakup, as defined by continuous elongation, or the absence of a steady drop shape. More recently, Bentley and Leal experimentally studied drops in steady linear flows over a wide range of conditions and showed that planar flows dominated by extension are most efficient in deforming drops; the addition of vorticity reduces elongation by rotating drops away from the principal straining axis. Bentley and Leal also determined that for near-critical flow strengths (for which drop deformation is no longer small), higher-order small deformation theory under-
estimates deformation and predicts a physical drop shape. Nevertheless, the theory is fairly accurate in estimating critical Capillary numbers, i.e., when viscous forces overcome surface tension forces, resulting in continuous drop elongation.

In terms of actual drop breakup during the application of a steady flow, fragmentation through capillary wave instabilities has been observed for drops being elongated into thin fluid threads.12–14 The process depends on the growth of disturbances on the thread surface; variations in thread radius create capillary pressure gradients which eventually shatter the thread into a large number of drops of similar size. Breakup during the application of a steady linear flow is a fairly extreme process that requires a drop length greater than thirty times the initial drop radius.

In contrast, breakup of much shorter drops can be induced by the sudden cessation of flow.15 After elongation by steady linear flows above a critical strength, drops have been observed to break upon cessation of the imposed flow.10,11,14,16 and Stone et al.15 named this deterministic breakup mechanism “end-pinching.” For a drop extended to a length greater than a critical value,15 the drop ends become bulb-like in the absence of an imposed flow and retract towards the drop center. At the same time, neck regions form near the ends and drain prior to complete retraction, thereby resulting in two or more daughter drops.17 Note that in general, the end-pinching mechanism generates larger (and fewer) daughter drops compared to the capillary wave instabilities, and therefore, differentiation between the two processes is important.12 Stone and Leal18 also studied step changes in steady linear flows, from supercritical to subcritical strengths, and depending on drop extension at the time of the step change, drops were observed to breakup via end-pinching while extending in the subcritical flow.18

In addition to relatively simple flow histories (i.e., step changes), a number of studies have been conducted to understand drop dynamics within more complex flows. Tjahjadi and Ottino19 placed drops in chaotic flows created between two eccentric rotating cylinders. Due to a positive mean strain rate, drops extended into long filaments (with lengths of hundreds of initial drop radii). Even though end-pinching was observed, capillary wave instabilities played the dominant role in breakup, and consequently, the threads eventually shattered into very small droplets.19 More recently, Cristini and Loewenberg20 used boundary integral methods to study drop breakup in isotropic turbulence. They observed significant drop elongation in particular regions of the flow; some of these extended drops ruptured through end-pinching, while others successfully relaxed to more stable shapes.20

The fiber bed flows in this work provide a model system since the problem of drop breakup can be studied independently in both simulation and experiment. In addition, this model flow provides an interesting situation where the mean flow field simply results in drop translation; fluctuations about the mean are responsible for drop deformation and potential breakup. Previously, Mosler and Shaqfeh7,8 studied this system using small deformation theory and emulsion experiments. Based on diluteness of the fiber bed, they considered the disturbance velocities far from any single fiber (i.e., at distances described by the pore size) and showed that small deformation theory predicts breakup within this Lagrangian-unsteady flow field. The researchers also concluded that the means by which the drop samples the unsteady flow must be important in the breakup process since the flow field type and strength are both ordinary at the time of breakup.8 More recently, we simulated drop migration through these same far-field flows using a boundary integral method and found drop breakup via end-pinching after bursts of uniaxial extensional flow.21 In terms of experimental work, Mosler and Shaqfeh7 used linear conservative dichroism and turbidity measurements to monitor deformation and drop sizes within emulsions passing through an experimental fiber bed. They observed an increase in deformation and breakup relative to their simulations, but the analysis was complicated by polydispersity and the inability to monitor individual drops.7

In this work, we consider the near-field interactions, which are important when a drop and a single fiber in the dilute bed are close (i.e., at a distance on the order of the fiber radius). After formulating the problem of interest in Sec. II, we describe in Sec. III an experiment for the direct observation of breakup mechanisms due to flow through a dilute fiber bed, thus allowing us to evaluate the role of near- and far-field interactions in drop breakup.8,21 After discussing the observed breakup mechanisms, we also present the breakup probability through the experimental fiber bed as a function of a Capillary number. In Sec. IV, we use boundary integral simulation to better understand the breakup processes resulting from near-field interactions and then attempt to achieve quantitative agreement between numerical predictions of the breakup probability and the experimental measurements. Finally, we conclude and comment on future work in Sec. V.

II. PROBLEM FORMULATION

We assume creeping flow conditions (Re=0) such that the mean, or superficial, velocity $U$ through an unbound porous medium is uniform, steady, and governed by Darcy’s law. Accordingly, the permeability $k$ describes the medium’s resistance to fluid flow, and the square-root of the permeability $k^{1/2}$ is the pore size, or screening length. Following previous work on drop breakup in the far field,8,21 the fiber bed is assumed to be dilute (solids fraction $\phi<1$) such that the pore size $k^{1/2}$ is much larger than the fiber radius $a_{fb}$. Consequently, the flow in regions far from any fiber varies on the length scale of the pore size, while near-field interactions are important when the separation between a drop and fiber is on the order of the fiber radius $a_{fb}$. The fiber bed of interest is disordered with respect to fiber position (i.e., fiber centers are randomly positioned), and the bed geometry is square-symmetric with fiber axes perpendicular to the mean velocity.

The drop fluid and suspending fluid are considered to be Newtonian with equal viscosities $\mu$. The two fluids are immiscible, surfactants are absent from the system, and interfacial energies are governed by the constant surface tension $\sigma$. 

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Note that we neglect gravitational effects by assuming the fluid densities are equal. The drop radius \( a \) is assumed small relative to the pore size.\(^{8,21}\) Since we consider a dilute emulsion (with negligible drop–drop interactions), deformation and breakup are functions of only the Lagrangian unsteady flow experienced by drops as they are carried through the fiber bed.

III. SINGLE-DROP EXPERIMENTS

A. Experimental apparatus and fluid properties

Construction of the fiber bed used in this study involved building a rectangular acrylic channel 90 cm in length with a square cross section 20 cm wide (interior dimension). The apparatus was designed for fluid flow along the bed length, and through a 50 cm section of the channel, we fixed rigid acrylic fibers with \( a_{fib} = 0.08 \) cm between the channel walls at random positions. Each fiber was oriented in one of the two directions parallel to the channel walls, and the overall bed geometry was square symmetric. A total of 199 fibers was placed through the 50 cm long section of the channel, resulting in a solids fraction \( \phi \) of 0.004.

We note that previous computational studies of drop breakup including only far-field interactions\(^{8,21}\) involved disturbance velocity statistics for a different bed geometry—one with isotropic fiber orientation. To justify direct comparison between these previous results and our experimental observations, we cite work on particle orientation\(^1\) and polymer conformation\(^7\) in which experimental observations using a square-symmetric fiber bed compared well with theoretical predictions based on isotropic fiber orientation.

For dilute beds with fiber axes normal to the mean velocity, the pore size is independent of fiber orientation in the plane perpendicular to the mean flow, as a first approximation. To estimate the dimensionless pore size \( \alpha = \kappa^{1/2}/a_{fib} \) under these conditions, we use the following relationship:\(^{22,23}\)

\[
\frac{\kappa^{1/2}}{a_{fib}} \approx \sqrt{\frac{\ln(1/\phi)}{8\phi}}.
\]

For \( \phi = 0.004 \), we calculate \( \alpha = \kappa^{1/2}/a_{fib} \approx 13 \) and \( \kappa^{1/2} \approx 1.0 \) cm, and the dimensions of the experimental fiber bed were based on this estimate. The length of the fiber bed was designed for 50 pore sizes, comparable to the length of the trajectories modeled in previous simulations.\(^{8,21}\) Based on transverse diffusivities for particle motion due to the far-field interactions in dilute isotropic beds,\(^7\) the channel was constructed with a half-width of \( 10\kappa^{1/2} \) to avoid drop migration close to the walls. Note that the pore size differs only slightly in dilute isotropic fiber beds;\(^{22,23}\) \( \alpha \approx 14 \) for \( \phi = 0.004 \).

The experimental fiber bed was filled with polybutene (Parapol 950, ExxonMobil Chemical Company), and the suspending fluid was circulated through the fiber bed as shown in Fig. 1 with a progressing cavity pump (model 71205 NNV, Roper Pump Company) driven by a 0.75 hp motor. This particular pump was selected for its ability to deliver pulseless flow of highly viscous fluids with minimal suction-side pressure, and the flow rates delivered by the pump were measured by simply monitoring the transported fluid volume over time. Superficial velocities \( U \) through the fiber bed could be varied between 0.0042 cm/s and 0.039 cm/s. For the largest velocity, we calculate the Reynolds number based on channel width as 0.003, confirming that inertia forces were negligible in these fiber bed flows.

The drop fluid used in our experiments was a polydimethylsiloxane (Rhodorsil 47V30000, Rhodia), and since the largest characteristic shear rate \( (Ua_{fib})/\kappa^{1/2} \) in these fiber bed flows was only 0.5 s\(^{-1}\), both fluids were assumed to be Newtonian based on previous rheological characterization.\(^{24,25}\) Using a cone-and-plate rheometer, the shear viscosity at room conditions was measured as 26 Pa s and 21 Pa s for the drop fluid and suspending fluid, respectively. Since the viscosity of each fluid remained constant up to at least a shear rate of 10 s\(^{-1}\), we estimate a constant viscosity ratio (drop fluid over suspending fluid) of 1.2 for our experiments.

To measure the interfacial tension between the two fluids, we followed a procedure analogous to that outlined by Tjahjadi et al.\(^{26}\) We monitored the relaxation of symmetrically deformed drops in a stagnant fluid, and our experimental measurements of length versus time were matched to boundary integral simulations by varying the drop relaxation time \( (\mu a/\sigma) \). In this way, we determined the surface tension \( \sigma \) as 1.1 ± 0.1 mN/m between the two fluids in this work. Note that surface tension generally decreases with molecular weight in polymer blends;\(^{27}\) thus our measurement is consistent with the larger surface tension values measured between higher molecular weight fluids from the same families by previous researchers.\(^{24,25}\)

To experimentally observe drop breakup within the fiber bed, individual drops were inserted into the suspending fluid using a needle, as shown in Fig. 1. Placement of the drop was varied along one direction, such that drops were positioned between 6.3 cm and 10.2 cm from the inside of the channel wall with the insertion port. The radius of each drop was measured upstream of the fiber bed by capturing images of the drop in the same field of view as the needle of known

![FIG. 1. Diagram of experimental apparatus showing circulation of suspending fluid, location for drop insertion, and orientation of video cameras.](image-url)
Our procedure for creating and injecting a single drop into the channel was reproducible in terms of drop size. In our experiments, we considered a single drop size (i.e., \( a = 0.11 \text{ cm} \)) for which \( a/\varepsilon^{1/2} = 0.1 < 1 \). Since we were not able to directly measure the pore size within the fiber bed, we scale drop radius with the fiber radius and fix the dimensionless drop size \( \beta (=a/a_{\text{fib}}) \) to a value of 1.4.

After a single drop was placed upstream of the fiber bed, it was carried through the array of fibers at a certain Capillary number representing the ratio of viscous forces to surface tension forces. The Capillary number we use to characterize the experiments is based on the fiber radius:

\[
Ca = \frac{\mu U a}{\sigma a_{\text{fib}}},
\]

where \( \mu \) is the viscosity of the suspending fluid. Note that the drop relaxation time \( \mu a/\sigma \) was approximately 20 s, as measured by the procedure described previously. The Capillary number was varied by changing the superficial velocity \( U \), and based on the physical parameters discussed here, we accessed \( Ca \) between 1 and 10 within our experiments.

For the two fluids in this study, the density difference is 0.09 g/cm\(^2\), and we calculate the Bond number\(^{28} \) as near unity for our experiments. Even though the Bond number is not small, we neglected gravitational effects since relatively large Bond numbers are necessary for gravity to destabilize drop shapes.\(^{28-30} \) In addition, drop shapes with either long tails or cavities, which are characteristic of buoyancy-driven instabilities,\(^{28-30} \) were not observed during our experiments. Finally, the translational velocity induced by the density difference for an undeformed drop\(^{28} \) was less than one third of the smallest superficial velocity in our study, and thus, gravity-induced settling was not important in our experiments.

As each drop was driven through the fiber bed, we captured images of the process using two black-and-white CCD video cameras (model XC-ST70, Sony Corporation) oriented in orthogonal directions, as shown in Fig. 1. By fitting each camera with a Fujinon CCTV lens (1:1.8/75, model CF75A) and a 20-mm extension ring, we viewed an area approximately 2 cm wide at the center of the channel. The signals from each camera were fed into a video cassette recorder and recorded on videotape. Our apparatus was designed such that the fibers were essentially transparent within the fluid; acrylic (polymethylmethacrylate) has an index of refraction close to that for polybutene. On the other hand, the index of refraction for polydimethylsiloxane is sufficiently different such that drop deformation was viewed without the use of dyes; imaging was enhanced with back lighting. Since drop migration through the fiber bed was relatively slow (from 20 minutes to more than 3 hours depending on \( Ca \)), each camera was mounted to a tripod, and the tripod heights were adjusted manually to maintain the translating drop within the field of view. Finally, drop images were analyzed using an Apple computer with a frame grabber board (Scion Corporation) and ScionImage software.

Our experimental observations include images of drop shape evolution as drops were carried through the fiber bed. We studied drop dynamics at four Capillary numbers (\( Ca = 1.1, 2.3, 5.0, \) and 10.0) and a single drop size ratio (\( \beta = 1.4 \)). Considering the dilution of the fiber bed (\( \phi = 0.004 \)), we were somewhat surprised to find that the majority of drops broke during their migration through the bed and that all breakup events resulted from near-field interactions with individual fibers. At the beginning of Sec. IV, we comment on the apparent discrepancy between our observations and previous work that predicted breakup in the far field.\(^{8,21} \) In this section, we describe the breakup mechanisms resulting from near interactions (which we name “graze” and “hairpin” processes) and present the probability of breakup measured during experiments as a function of \( Ca \).

In Fig. 2, we provide images of a drop undergoing breakup by the “graze” mechanism. Note that the uniform flow velocity convects fluid from top to bottom in all images from our experiments. In this particular example, the drop passed within close proximity to a single fiber in the medium, as seen in the first two images. Even though the fibers were transparent within the suspending fluid, the match in index of refraction is imperfect, and thus, fiber surfaces can be delineated in images. The drop deformed due to its close interactions with the fiber, and elongation continued as the drop was carried downstream. The extended drop is asymmetric with more fluid collected at the downstream end, and after the extension process, the two ends of the drop formed spherical shapes and attempted to retract towards the center of mass. The pinch-off process competes with relaxation, and we found that the downstream neck drained first, resulting in
breakup of the larger bulb-like end. The daughter drops were carried through the fiber bed and interacted with other fiber elements, but to simplify our analysis, we only considered the first breakup event for each drop. We note that the graze mechanism is an asymmetric version of the end-pinching process observed by Stone et al. during the relaxation of symmetric elongated drops.

By analyzing drop images, we quantified lengths by measuring the largest dimension of each drop shape relative to the initial drop size \(a\). In Fig. 3, we plot drop length as a function of time for the drop described in Fig. 2. After quantifying length evolution, we found that drop extension reached a maximum of approximately 8\(a\) before retraction and subsequent pinch-off. Note that this value is comparable to the critical elongation ratio measured by Stone et al. for breakup during relaxation of a symmetrically extended drop.

In our single-drop experiments, we also observed graze breakup in which drops exhibited very large lengths, greater than the field of view (\(\approx 20a\)). Pinch-off occurred first next to the smaller upstream end, and this initial break was sometimes followed by additional end-pinching events at the upstream end. Eventually, a neck next to the larger downstream end formed and drained, resulting in the largest daughter drop. Furthermore, our experiments showed that graze breakup was inhibited at small Ca by strong surface tension forces that prevented large drop extensions; this trend is quantified later in this section during our discussion of breakup probabilities.

In addition to drop breakup by the graze mechanism, we also observed breakup via a "hairpin process." In Fig. 4, we present a sequence of images showing the formation of a hairpin, or inverted "U," as a drop at Ca = 2.3 flowed past a fiber. Note that asymmetric hairpins were most frequent due to misalignment between the drop and fiber centers. After formation of the hairpin, the arms continued to flow downstream as the drop fluid was carried by viscous forces; at this point, surface tension forces were unable to prevent further elongation. As shown in the last image, a thin thread of fluid remained upstream of the fiber and moved very close to the fiber as fluid drained into the hairpin arms, or strands; we never saw instances of the drop fluid wetting the fiber surface. Actual breakup of the hairpin involved the ends of the drop first breaking through the end-pinching process; for the particular run shown in Fig. 4, the length of each strand was approximately 50\(a\) at pinch-off. After the end drops detached from each strand, two thin threads of fluid remained upstream of the newly formed daughter drops and eventually fragmented through capillary wave instabilities. The daughter drops resulting from capillary waves formed in equal-size pairs, possibly due to hydrodynamic interactions between the two threads during the breakup process. Since the thickness of the threads decreased closer to the fiber, the smallest daughter drops were formed near the fiber. Finally, we point out that Stone and Leal showed that the time scale for capillary wave instabilities is larger than that for end-pinching, and our experimental observations are consistent with their findings.

Regarding the effect of Ca on breakup by the hairpin process, we observed less elongation of the strands prior to pinch-off as the Capillary number was decreased. At Ca = 1.1, breakup at the ends of the hairpin occurred at a relatively short strand length, i.e., less than 20\(a\). On the other hand, strands were typically longer than 100\(a\) before breakup at Ca = 10. The time scale for breakup appears to depend on surface tension and thus remains constant over the range of Capillary numbers, while the length of the hairpin configuration at breakup is governed by the magnitude of the superficial velocity. Therefore, as Ca is reduced, slower veloci-
ties result in less extension of the hairpin, and fewer daughter drops are created upon completion of the breakup process for the less extended drops.

Taking data from approximately 20 drops at each flow rate, we quantify breakup probabilities by plotting the percentage of drops that break as a function of Ca in Fig. 5. We also include contributions from the graze mechanism to the total. As expected, a greater number of breakup events was observed for larger Ca since stronger viscous forces increase drop extension. In addition, we find that hairpin breakups accounted for almost all of the breakup events near Ca=1; strong surface tension prevents the large drop deformations necessary for the graze mechanism.

IV. BOUNDARY INTEGRAL SIMULATION

A. Methodology

As discussed in Sec. I, previous computational work on drop breakup in flows through fixed fiber beds has focused on the far-field interactions. Far-field breakup rates as a function of the Capillary number in a more concentrated bed (Φ = 0.025) have been predicted using both small deformation theory and boundary integral simulation, but during experiments, we did not observe a single instance of breakup in the far field. Note that far-field breakup rates for the solids fraction studied in this work (i.e., Φ = 0.004) have not been predicted and the Gaussian flow statistics used in the previous numerical studies have not been experimentally verified.

In this section, we focus on the near-field interactions (at distances on the order of the fiber radius a_{fib}) and couple a boundary integral technique with a description of the flow around a single fiber in a porous medium. Even though our experiments were conducted at a viscosity ratio of 1.2, previous work on steady flows has shown that characteristics of stable deformed drops along with critical conditions for breakup are weak functions of viscosity ratio, and consequently, we assume a viscosity ratio of unity to simplify the boundary integral representation. As a result, the interfacial velocity u_i at point x^0 on the drop is a sum of the imposed flow field u^e_i and an integral that describes flow induced by surface tension forces:

\[ u_i(x^0) = u_i^e(x^0) - \frac{1}{Ca} \int_S n_j(x) \frac{\partial n_j(x)}{\partial x_k} G_{ij}(x - x^0) dS \]

where the integration variable x is a vector from the drop center to an arbitrary point on the drop surface. The integrand involves the outward unit normal n_j(x), local surface curvature \( \frac{\partial n_j(x)}{\partial x_k} \), and the Stokeslet \( G_{ij} \). Note that even though the Green’s function for a point force within a Brinkman medium has been developed, use of the free-space form is justified due to the parameter regime of interest in this study, i.e., \( \kappa^{1/2} > a \). In Eq. (3), all lengths have been scaled with the equilibrium drop radius a and velocities with \( (U/a_{fib}) u \) where \( U/a_{fib} \) is one measure of the velocity gradient near a fiber. Finally, after evaluating the interfacial velocities, the drop shape is evolved forward in time by utilizing a Lagrangian representation of the kinematic condition.

The three-dimensional boundary integral simulations used in this study were developed by Loewenberg and coworkers, and since details on the numerical implementation are available elsewhere, only a brief summary follows. Drop shapes are described by a mesh of triangular elements, ranging from 100 to 200 elements for the initial spherical shape. To evaluate Eq. (3), the surface integral is discretized, and the integrand is evaluated at triangle vertices. A singularity subtraction technique is used to evaluate the integrand when \( x = x_i \). and a local surface-fitting method provides accurate calculation of normal vectors and surface curvature. When evoking the drop interface forward in time, an explicit second-order scheme is used, along with an adaptive time step that depends on the minimum distance between grid points on the drop. The simulation developed by Loewenberg and co-workers also features an adaptive grid that maintains high accuracy by adding elements in regions with high curvature. Finally, numerical implementation of the boundary integral equation must also include an algorithm to capture the drop breakup process. Since the radius of neck regions decreases linearly with time just prior to pinch-off, a drop is considered to breakup during simulation if this linear decrease in neck radius is observed after the neck radius is less than a threshold value of 0.1a.

To consider near-field interactions, the imposed velocity \( u_i^e \) in Eq. (3) is set to the two-dimensional field describing flow around an infinitely long fiber in an isotropic porous medium, as derived by Spielman and Goren from Brinkman’s equation. Note this approach is just a first approximation; we assume the disturbance field around the fiber is unperturbed by the presence of the drop and that the drop hydrodynamically interacts with the fiber only through this idealized field. In addition, we emphasize that the near-field flow around an individual fiber is dependent on the pore size, or the hydrodynamic screening from the surrounding matrix of fibers. The strength of the viscous forces around the fiber is a function of \( \alpha \), the ratio of pore size \( \kappa^{1/2} \) to fiber radius \( a_{fib} \), and velocity gradients along the fiber surface decrease with increasing pore size, like \( 1/ln \alpha \). Since drop size was comparable to fiber radius during our experiments (i.e., \( \beta = 1.4 \)), the flow field is evaluated at each grid point on the drop surface such that drop experiences a nonlinear flow as it migrates past a fiber. Since we consider three-dimensional drops, the imposed velocity along the fiber axis is set to zero.

In these near-field simulations, we first place an undeformed drop a distance of 50a_{fib} upstream of the fiber center. Displacement in the orthogonal direction is defined as the offset e, such that the initial, drop center-of-mass position (in units of a_{fib}) is \( x^{com} = (-50, e, 0) \), relative to the fiber center at (0,0). After using Eq. (3) to calculate interfacial velocities generated by the imposed flow field and surface tension, the drop shape is advanced forward in time, and a new center-of-mass position is calculated. When a drop is close to the fiber, velocities generated by surface tension can create an artificial situation in our model where regions of the drop penetrate the fiber surface. To alleviate this problem, we add a repulsive term to the local surface curvature, as implemented by Loewenberg and Hinch to simulate drop shapes in emulsions under shear.
length is observed due to drop relaxation—a result of alignment in the flow direction during a local shear flow. Downstream of the fiber, the drop elongates again due to the extensional character of the flow field and remains aligned with the mean flow direction. The maximum drop length occurs at approximately 8a_{ib} from the fiber, and far downstream where velocity gradients are reduced, surface tension forces cause the drop to relax to the equilibrium shape. Note that for this set of parameters, relaxation of drop shape is gradual and occurs over more than 30a_{ib} downstream of the fiber even though overall extension is not large. Regardless, modeling the breakup process as a single collision remains valid; during experiments, a pre-extended drop approaching a second fiber would first retract to a near-equilibrium configuration due to the compressive flow along the mean velocity direction.

In Fig. 8, we plot drop lengths corresponding to trajectories with offsets ranging from 0.55 to 1.0. As the offset \( \varepsilon \) is reduced, drop elongation increases in a nonlinear fashion. Drops slightly closer to the fiber surface sample larger velocity gradients due to large changes in flow strength with respect to position. Also, drops closer to the fiber experience strong viscous forces over longer periods of time since the velocity in the mean flow direction is reduced near the fiber surface. Finally, drop relaxation time increases with drop ex-

\begin{equation}
 h(x) = \frac{\partial n_f(x)}{\partial x_k} + \frac{\exp[-d(x)/0.1]}{1 - \exp[-d(x)/0.1]},
\end{equation}

where \( d(x) \) is the distance (in units of \( a_{ib} \)) between the drop surface point located at \( x \) and the fiber surface; \( h(x) \) replaces the surface curvature in Eq. (3). For all of the work presented here, the repulsive shell was restricted to a distance of 0.1a_{ib} from the fiber surface. We also tested a thinner shell (0.05a_{ib}) and found that breakup mechanisms and probabilities were unchanged.

We did consider the effect of the no-slip fiber surface on the interfacial velocities induced by surface tension. First, the fiber surface was discretized using an adaptive grid that increased the density of elements in regions of the fiber close to the drop. Next, we calculated the surface traction, or force density, that resulted on the fiber due to the surface tension forces acting on the drop. Finally, the surface tractions on the fiber were reflected on the drop to contribute to the interfacial velocity. Our technique essentially corrected the velocities induced by surface tension to account for the no-slip condition on the fiber surface, but the consideration of these tractions had little influence on the results presented in this study. The presence of the fiber does weaken surface tension forces and therefore increases deformation, but the effect is not sufficient to change breakup mechanisms and the critical conditions for breakup. Therefore, in this work, we neglect the fiber surface tractions induced by surface tension and refer to a more detailed discussion available elsewhere.\(^{21}\)

B. Comparisons with experiments

1. Breakup mechanisms

To illustrate the breakup mechanisms predicted by simulation of drops flowing through the near field, we vary the initial offset \( \varepsilon \) in this section while fixing the other parameters (\( Ca = 3, \alpha = 5, \beta = 1 \)) to values comparable to the experimental conditions. In the next section, we describe how breakup in the near field varies with Capillary number and pore size at \( \beta = 1.4 \). Note that due to the symmetry of the flow field, we consider only positive values of the initial offset.

In Fig. 6, we describe the center-of-mass trajectory along with length and orientation evolution for \( \varepsilon = 1.0 \); drop shapes corresponding to this example are presented in Fig. 7. Note that length is defined as the largest distance between two surface grid points while orientation is quantified by the angle between the vector describing the drop length and the direction of the mean velocity (positive \( x_2 \)). We find that the drop center follows a path identical to the trajectory of a point particle initially placed at \( x_1 = -50 \) and \( x_2 = 1 \). Approximately 10a_{ib} upstream of the fiber, the drop experiences a compressional flow along \( x_1 \) and consequently, extends while oriented in the \( x_2 \) direction. Deformation increases as the drop is carried closer to the fiber, and around 5a_{ib} upstream of the fiber, the drop begins to experience a shear flow and rotates towards the mean flow direction. As the drop passes by the fiber (\( x_1 = 0 \)), extensional and compressional forces become minimal, and a reduction in drop length is observed due to drop relaxation—a result of alignment in the flow direction during a local shear flow. Downstream of the fiber, the drop elongates again due to the extensional character of the flow field and remains aligned with the mean flow direction. The maximum drop length occurs at approximately 8a_{ib} from the fiber, and far downstream where velocity gradients are reduced, surface tension forces cause the drop to relax to the equilibrium shape. Note that for this set of parameters, relaxation of drop shape is gradual and occurs over more than 30a_{ib} downstream of the fiber even though overall extension is not large. Regardless, modeling the breakup process as a single collision remains valid; during experiments, a pre-extended drop approaching a second fiber would first retract to a near-equilibrium configuration due to the compressive flow along the mean velocity direction.

In Fig. 8, we plot drop lengths corresponding to trajectories with offsets ranging from 0.55 to 1.0. As the offset \( \varepsilon \) is reduced, drop elongation increases in a nonlinear fashion. Drops slightly closer to the fiber surface sample larger velocity gradients due to large changes in flow strength with respect to position. Also, drops closer to the fiber experience strong viscous forces over longer periods of time since the velocity in the mean flow direction is reduced near the fiber surface. Finally, drop relaxation time increases with drop ex-

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{Fig_6.png}
\caption{Center-of-mass trajectory along with length and orientation for drop that passes by fiber and remains stable (\( Ca=3, \alpha=5, \beta=1, \varepsilon=1 \)).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{Fig_7.png}
\caption{Shapes during lifetime of stable drop described in Fig. 6.}
\end{figure}
tension; elongated drops are more easily deformed further. In addition to large increases in extension, drops that pass closer to the fiber surface do not exhibit relaxation in length near \( x_1 = 0 \), probably due to the increase in relaxation time with drop length and/or simultaneous sampling of shear and extensional flows.

For \( \text{Ca} = 3, \alpha = 5, \) and \( \beta = 1 \), we find that an offset of 0.6 results in significant elongation but that the drop completely relaxes far downstream of the fiber. In contrast, the case with \( \varepsilon = 0.55 \) results in drop breakup; pinch-off is observed as the extended drop attempts to relax, as shown in Fig. 9. Consequently, for these conditions (\( \text{Ca} = 3, \alpha = 5, \beta = 1 \)), we estimate the critical offset for breakup \( \varepsilon_{\text{crit}} \) as \( 0.575 \pm 0.025 \). Regarding the breakup event, the entire drop passes the fiber on one side, in a manner completely analogous to the graze mechanism observed during experiments. The pinch-off process is comparable to the end-pinching mechanism observed and explained by Stone et al.\(^{15}\) in their work on drop breakup after the cessation of steady, linear flows. In addition, they defined a critical maximum length necessary for breakup, and drop lengths in Fig. 8 suggest a critical length between 7.5 and 10 initial drop radii. Recall that the example of the graze breakup presented in the experimental section exhibited a maximum length of \( 8a \).

As observed during experiments, extended drops from the near-field simulations are highly asymmetric as shown in Fig. 9. Due to the extensional flow downstream of the fiber, fluid collects at the downstream end of the drop. As the drop flows downstream of the fiber, velocity gradients decay, and surface tension creates bulbous ends that differ greatly in size because of the initial asymmetric drop shape. Note that due to greater curvature and consequently stronger surface tension forces, a spherical end forms first on the upstream side of the drop. After flowing into the uniform velocity field far downstream of the fiber, the extended drop attempts to relax, and competition between the retraction and pinch-off processes determines whether the drop breaks. For pinch-off close to \( \varepsilon_{\text{crit}} \), we find that the neck region adjacent to the downstream end drains first, as was observed during experiments. We monitor the neck drainage process by plotting surface curvature \( \partial n_1 / \partial x_1 \) along the drop, relative to the center-of-mass, as shown in Fig. 10. At the beginning of the retraction process, the upstream end exhibits a larger local minimum in curvature and thus, is well-positioned for neck formation. The presence of this local minimum in curvature results in a capillary pressure gradient that drives fluid into the minimum, and the resulting mass flux creates a neck region characterized by a local maximum in curvature.\(^{15,17}\) But, relaxation of the upstream end is faster due to greater surface curvature and smaller size, and this retraction process appears to hinder drainage of the upstream neck. In contrast, retraction of the larger downstream end is relatively slow and thereby, allows neck drainage to proceed to completion.

As the offset \( \varepsilon \) is reduced further below the critical value, drop breakup via the graze mechanism is predicted with pinching first at the upstream end of the drop. For \( \varepsilon \)...
=0.5, the drop exhibits greater elongation as shown in Fig. 11 and consequently, continues to extend at further distances downstream of the fiber. As with breakup for ε near ε\text{crit}, a bulbous cap forms first at the upstream end as shown in Fig. 12. Note that the bulbous end forms even while the entire drop is extending, probably a result of greater curvature at this end and the weak nature of the imposed flow. Formation of the spherical end sets up the initial capillary pressure profile (local minimum in curvature) for the end-pinching process as shown in Fig. 13. In contrast to near-critical graze breakup, neck formation and drainage begin while the drop is extending. The extension process appears to offer less hindrance to the neck drainage process, and as observed during experiments, the upstream end pinches first, forming a relatively small daughter drop. Note that the pinch-off process for graze breakup at ε<ε\text{crit} is similar to observations made by Stone and Leal\textsuperscript{18} on breakup resulting from reductions in flow strength; they also observed neck formation and drainage during global elongation. Following the initial pinch-off, we suspect the dynamics observed during near-critical breakup will result in the formation of a large daughter drop from the downstream end. Regardless of the initial pinch-off location, we characterize all graze breakup by a drop that becomes highly extended after flowing past a single side of the fiber surface. Either during retraction or at the end of the extension process, actual breakup results from an asymmetric end-pinching process.

For initial offsets closer to zero, we predict the second breakup process, as shown in Fig. 14 for ε=0.3 with Ca = 3, α=5, and β=1. As observed during experiments, the hairpin mechanism is characterized by a drop flowing past a fiber with portions of the drop on opposite sides of the fiber. Prior to hairpin formation, the compressional flow upstream of the fiber (i.e., extension along x_2) overcomes surface tension forces, and as a result, the diverging streamlines carry drop fluid on opposite sides of the fiber. After drop fluid is carried into the region defined by x_1>0 and x_2<0, neither arm is able to retract. Surface tension forces along the cylindrical region of the drop upstream of the fiber are unable to counteract the viscous forces, in a manner analogous to continuous drop elongation in a strong steady extensional flow. During simulation, large velocities are induced by surface tension in the mean velocity direction as the hairpin configuration extends with time. These velocities require very small time steps to prevent penetration through the repulsive shell; to minimize computation times, we cease the simulation after an arm is pulled into the quadrant defined by x_1>0 and x_2<0 since breakup is inevitable. Note that the hairpin configuration becomes more symmetric as ε→0.

In this section, we have shown that the drop breakup mechanisms observed during experiments are qualitatively captured through boundary integral simulation of near-field interactions with individual fibers in a porous medium. We also defined a critical offset ε\text{crit} that can be calculated to describe the onset of breakup, and in the next section, we evaluate ε\text{crit} as a function of the Capillary number Ca to compare with the experimentally observed breakup probabilities.

2. Breakup probabilities

In Fig. 15, critical offsets ε\text{crit} are plotted as a function of the fiber-size Capillary number Ca and dimensionless pore size α for β=1.4. Note that the error bars represent a constant uncertainty of ±0.025 for all critical offsets. As expected, the critical offset for breakup increases with Capillary number; viscous forces are stronger for larger values for Ca and result in greater drop extension. In addition, since the strength of velocity gradients increases for smaller pore sizes, drops sample regions of stronger flow and breakup at
larger $\varepsilon$ as $\alpha$ is reduced. Regarding the critical offset as a function of dimensionless drop size $\beta$, this relationship is important for understanding the breakup of daughter drops formed by an initial breakup event and is briefly discussed elsewhere.$^{21}$

Relative contributions from the hairpin and graze mechanisms are determined by calculating critical offsets for hairpin formation. For offsets below the critical value for breakup, we monitor the breakup mechanism to find the offset that describes the transition from hairpin to graze breakups. The critical offset for hairpin formation is plotted versus Ca for $\alpha = 5$ and $\beta = 1.4$ at bottom of Fig. 15, and as suggested by our experimental observations, graze breakups are absent at small Ca. For Ca $\leq 1.0$, viscous forces are too weak to induce the large drop deformations necessary for the graze process. Under these conditions dominated by surface tension, drops remain stable even for small offsets, and all breakup events occur through the hairpin mechanism. Note that drops placed at $\varepsilon = 0$ always appear to break, regardless of Ca and $\alpha$. For large Ca, contributions from the graze mechanism are more significant but still secondary to the hairpin process.

To predict breakup probabilities using the critical offsets from boundary integral simulations, we assume a simple model for drop center-of-mass trajectories, that they follow straight lines through the fiber arrangement in the experimental bed. We first calculate breakup probabilities as a function of the critical offset $\varepsilon^{\text{crit}}$ by randomly selecting initial drop positions upstream of the fiber bed within a rectangular region (3.9 cm by 1 cm) equivalent to that used during experiments. The minimum distance between the straight line through an initial position and each of the 199 fibers is calculated, and if any of these distances is less than $\varepsilon^{\text{crit}}$, then the drop is assumed to break. After evaluating 10 000 initial drop positions for each critical offset, we calculate the percentage of breakup events, and the probability of breakup increases with $\varepsilon^{\text{crit}}$ as expected. In addition, the change in the probability decreases with the critical offset as the percentage of breakup events approaches 100% near $\varepsilon^{\text{crit}} = 1$. Essentially, our calculation considers the projection of all fibers (each with effective radius of $\varepsilon^{\text{crit}}$) on a plane perpendicular to the direction of the mean velocity, and the fractional area filled by the fiber projections within the rectangular area for initial drop placement is the probability of breakup. Consequently, for small $\varepsilon^{\text{crit}}$, an incremental increase in critical offset results in a large increase in this overlap area and therefore, a large increase in breakup probability. Finally, we note that even though the fiber bed is dilute, there are very few pores that run straight through the bed; an effective fiber radius of only $1.0a_{\text{fib}}$ ($\varepsilon^{\text{crit}} = 1$) results in breakup for almost all initial positions even though the pore size is much larger than the fiber radius.

After estimating the breakup probability as a function of critical offset, we relate the probability to physical parameters (Ca and $\alpha$) using Fig. 15. In Fig. 16, we present near-field predictions for percentages of drops that breakup through the experimental fiber bed as a function of Ca and $\alpha$ at $\beta = 1.4$. Note the error bars on our simulation results are determined using the $\pm 0.025$ uncertainty in the $\varepsilon^{\text{crit}}$ estimates. We find a large discrepancy between the predictions and experiment results near Ca $= 1$, and later in this section, we address possible explanations. The simulation results do follow the expected trends with respect to Ca and $\alpha$, and the near-field predictions for $\alpha = 5$ compare best with the experimental observations ($\alpha = 13$). Recall that we used Eq. (1) to estimate the mean pore size within the experimental bed. Since the local pore size around each fiber in the bed can vary and viscous forces around a fiber increase with decreasing pore size, breakup during experiments probably occurred around fibers with local pore sizes less than the mean. Therefore, the comparison in Fig. 16 suggests that $\alpha = 5$ is a better estimate of the relevant pore size (i.e., local pore size around fibers responsible for breakup) for our model of near-field breakups.

To complete the comparison between simulations and experiments, we consider contributions from the two breakup mechanisms by using the critical offsets for hairpin formation, shown in Fig. 15. During the calculation of offsets
between drop center and fiber centers, we begin with fibers at the top of the channel and proceed to the bottom. For an offset less than the \( \alpha \) for breakup, we compare the offset to the critical value for hairpin formation, and if the offset is larger, then the drop is considered to have broken through the graze mechanism. Predictions for contributions from the graze mechanism with \( \alpha = 5 \) are presented in Fig. 17 along with the experimental observations, and we find good agreement with both showing a reduction in graze breakups as \( \alpha \) is decreased. Nonetheless, the most striking part of this comparison is that the near-field predictions strongly underestimate the overall breakup probability near \( \alpha = 1 \), i.e., they underestimate the hairpin breakup events.

To evaluate sources for this discrepancy, we focused on our assumption of straight-line trajectories and allowed for drops to disperse in the directions orthogonal to the mean velocity. Even though drop breakup was controlled by near-field interactions, a significant portion of a drop’s lifetime was spent far from any single fiber, and consequently, we first considered the dispersion resulting from the far-field disturbance velocities. In addition, we studied stable interactions between drops and a fiber and predicted shifts in drop trajectories for offsets greater than critical at which the presence of the fiber forces the drop to follow a different set of streamlines. Results from both analyses are presented elsewhere.\(^{21}\) and here we simply note that neither model was able to provide quantitative agreement with the experimentally observed breakup probability near \( \alpha \). Shifts in drop trajectories due to stable interactions with individual fibers resulted in a slight increase in the breakup probability, and we suggest that at small Capillary numbers, hydrodynamic interactions between multiple fibers might steer or direct drops into fibers, thereby enhancing the probability of breakup. This is a subject of future work.

Finally, in addition to breakup probabilities, we also estimate breakup rates from the near-field simulations. By assuming straight-line trajectories, we calculate the number of surviving drops as a function of distance for each Capillary number and then fit each curve to an exponential decay. The breakup rate in terms of fraction of population that breaks per unit flow time ranges from \( 9 \times 10^{-5} \) to \( 2.5 \times 10^{-3} \) for \( 0.5 \leq \alpha \leq 5 \). Note that much larger breakup rates, i.e., \( 5 \times 10^{-3} \) for \( \alpha = 0.3 \), have been predicted for drop breakup in isotropic turbulence.\(^{20}\)

V. CONCLUSIONS

In this work, we have presented a thorough investigation of drop breakup in flows through dilute random beds of fixed fibers. Based on experimental observations, we discovered that near-interactions controlled the breakup process, while breakup in the far field was not observed. We also found that breakup was relatively frequent even though the experimental bed was dilute in terms of solids fraction; instead, projections of fibers on a plane orthogonal to the mean velocity are the controlling factor. During the experiments, we observed two types of breakup mechanisms, and these processes were also captured through boundary integral simulation of drops in an idealized flow field around a single fiber in a porous medium. The graze mechanism involved drops flowing closely past a fiber, extending greater than a critical length, and then breaking via an asymmetric end-pinning process. In addition to grazing breakup events, we also described a hairpin mechanism in which regions of the drop flow around both sides of a fiber, forming a characteristic hairpin configuration. Depending on the flow strength, the hairpin formation stretched in the mean velocity direction, the two ends broke through end-pinning, and the remaining threads of fluid eventually shattered through capillary wave instabilities. Both experiments and simulations showed inhibition of the graze mechanism as viscous forces were reduced relative to surface tension. Since each mechanism results in qualitatively different daughter drop sizes, changing the frequency of one mechanism over the other may be desirable and should be possible by varying the Capillary number. Finally, we were able to achieve quantitative agreement between numerical predictions and experimental observations of the breakup probability for conditions dominated by viscous forces. We found a significant discrepancy at small Capillary numbers which we attributed to our assumption of straight-line trajectories through the fiber bed.

With regards to broader implications of this work, porous media are used in industrial settings for mixing and filtration processes, and our results and conclusions should provide insight into related problems, specifically in terms of microstructure configurations near fixed obstacles. In addition, we have shown with our experimental work that even dilute fiber beds can be highly effective in inducing drop breakup; due to their high permeability, dilute beds also offer minimal pressure drop. Finally, we found that near-field interactions can actually dominate the evolution of microstructure in dilute beds. Even though drops in dilute fiber beds spend most of their lifetime far from individual fibers, they eventually encounter a single fiber, and this close interaction is critical in terms of drop breakup. In contrast to particle orientation and polymer stretch, drop breakup is an irreversible process, and so, these rare near interactions strongly influence emulsion properties (e.g., drop size).
In terms of future work on these fiber bed flows, the discrepancies between numerical predictions and experimental observations of drop breakup suggest a number of new problems. Characterization of the actual disturbance velocities within the experimental fiber bed would prove useful in evaluating the far-field Gaussian statistics developed by Shaqfeh and Koch and variations in local pore sizes relative to the mean value. In addition, our analysis of drop trajectories suggested the role of multiple-fiber interactions. We believe model problems can be developed to understand drop dynamics in velocity fields created by multiple fibers, and results should provide insight into the role of these interactions on our experimental observations.

Finally, the boundary integral simulations and experimental apparatus used in this work are robust tools for understanding a wide range of problems, including the effects of gravity and surfactants. Preliminary experimental studies have already been conducted with elastic and low-viscosity drop fluids, and observations suggest new dynamics including reduced elongation of the hairpin formation (elastic) and collision-avoiding behavior (low viscosity).

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